# Oklahoma Elementary Mathematics Specialist

A Certification Process 2011-2012

Oklahoma State Department of Education Oklahoma State Regents for Higher Education Oklahoma Commission for Teacher Preparation

### **Table of Contents**

A	Ce	rtifi	catio	n P	ro	cess
---	----	-------	-------	-----	----	------

Background	3
Program Prerequisites and Certification Requirements	4
<ul> <li>A. Program Prerequisites</li> <li>B. Certification Requirements</li> <li>OEMS Development Process</li> <li>OEMS Certification Examination Structure</li> </ul>	4 4 6 7
Pedagogical Content Knowledge for Teaching Mathematics	8
Section 1: Domains and Standards	8
<ul> <li>Domain I. Number and Operations: Content, Instruction, and Assessment</li> <li>Domain II. Algebra and Functions Content: Instruction, and Assessment</li> <li>Domain III. Geometry and Measurement</li> <li>Domain IV. Data Analysis and Probability</li> <li>Domain V. Mathematics Instructional Leadership</li> </ul>	8 9 9 10 11
Section 2: Competencies	12
Domain I: Number Concepts and Operations: Content, Instruction, and Assessment	
<ul> <li>Competency 001</li> <li>Competency 002</li> <li>Competency 003</li> <li>Competency 004</li> </ul> Domain II: Algebra and Functions: Content, Instruction, and Assessment	12 12 13 13
	4.5
<ul><li>Competency 005</li><li>Competency 006</li><li>Competency 007</li></ul>	15 15 16

<b>Domain III: Geometry and Measurement: Cont</b>	ent, Instruction, and Assessment
<ul><li>Competency 008</li><li>Competency 009</li></ul>	17 17
Competency 009     Competency 010	18
Competency 011	18
Domain IV: Data Analysis and Probability: Cont	ent, Instruction, and Assessment
Competency 012	20
<ul> <li>Competency 013</li> </ul>	20
<ul> <li>Competency 014</li> </ul>	20

• Competency 015 22

#### **Background**

The Association of Mathematics Teacher Educators (AMTE)<sup>1</sup>, the Association of State Supervisors of Mathematics (ASSM)<sup>2</sup>, the National Council of Supervisors of Mathematics (NCSM)<sup>3</sup>, and the National Council of Teachers of Mathematics (NCTM)<sup>4</sup> recommend the use of Elementary Mathematics Specialists (EMS) in PK–6 environments to enhance the teaching, learning, and assessing of mathematics in order to improve student achievement, and further advocate that every elementary school have access to an EMS. Districts, states/provinces, and higher education should work in collaboration to create: (1) advanced certification for EMS professionals; and (2) rigorous programs to prepare EMS professionals. EMS professionals need a deep and broad knowledge of mathematics content, expertise in using and helping others use effective instructional practices, and the ability to support efforts that help all PK–6 students learn important mathematics. Programs for EMS professionals should include foci on mathematics content knowledge, pedagogical knowledge, and leadership knowledge and skills.

Oklahoma, like most states, experiences challenges in recruiting and developing EMS professionals. In response, the Oklahoma Elementary Mathematics Specialist (OEMS) certification program has been proposed to address this need. Program preparation will include both coursework and a certification examination. Contact Saeed Sarani, Oklahoma State Regents for Higher Education, <a href="mailto:ssarani@osrhe.edu">ssarani@osrhe.edu</a>, or Courtney Lockridge, Oklahoma State Department of Education, <a href="mailto:Courtney.Lockridge@sde.ok.gov">Courtney.Lockridge@sde.ok.gov</a> for additional information.

<sup>&</sup>lt;sup>1</sup> Association of Mathematics Teachers Educators (AMTE) is the largest professional organization devoted to the improvement of mathematics teacher education—it includes over 1000 members devoted to the pre-service education and professional development of K-12 teachers of mathematics, <a href="http://www.amte.net/">http://www.amte.net/</a>.

<sup>&</sup>lt;sup>2</sup> The Association of State Supervisors of Mathematics is an organization whose members provide supervising and/or consulting services in the area of mathematics as a staff member of the education agency of any state in the United States of America, the District of Columbia, possessions of the United States of America, territory of the United States, US Department of Defense, Canadian Province, or Bermuda, <a href="http://www.assm.us/">http://www.assm.us/</a>.

<sup>&</sup>lt;sup>3</sup> The National Council of Supervisors of Mathematics (NCSM) is a mathematics leadership organization for educational leaders that provides professional learning opportunities necessary to support and sustain improved student achievement, <a href="http://www.mathedleadership.org/">http://www.mathedleadership.org/</a>.

<sup>&</sup>lt;sup>4</sup> The National Council of Teachers of Mathematics is a public voice of mathematics education supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development and research, <a href="http://www.nctm.org/">http://www.nctm.org/</a>.

#### **A Certification Process**

OEMS professionals must know and understand elementary school mathematics as well as how mathematics concepts and skills develop through the secondary level. This includes specialized knowledge that teachers need in order to understand and support student learning of elementary mathematics. OEMS professionals also need to be prepared to take on collegial non-evaluative leadership roles within their schools and districts. They must have a broad view of many aspects and resources needed to support and facilitate effective instruction and professional growth.

#### **Program Prerequisites and Certification Requirements**

#### A. Program Prerequisites:

**Prior to beginning coursework** to become an Oklahoma Elementary Mathematics Specialist, the following two prerequisites must be met:

- 1. Valid teacher certification in Elementary Education and/or Early Childhood Education; and
- 2. Two years of full-time teaching experience in grades **Pre-Kindergarten through Grade 5** at an accredited school under a valid state issued teacher credential

#### **B. Certification Requirements:**

In order to obtain certification in Oklahoma as an Elementary Mathematics Specialist, the following requirements must be fulfilled:

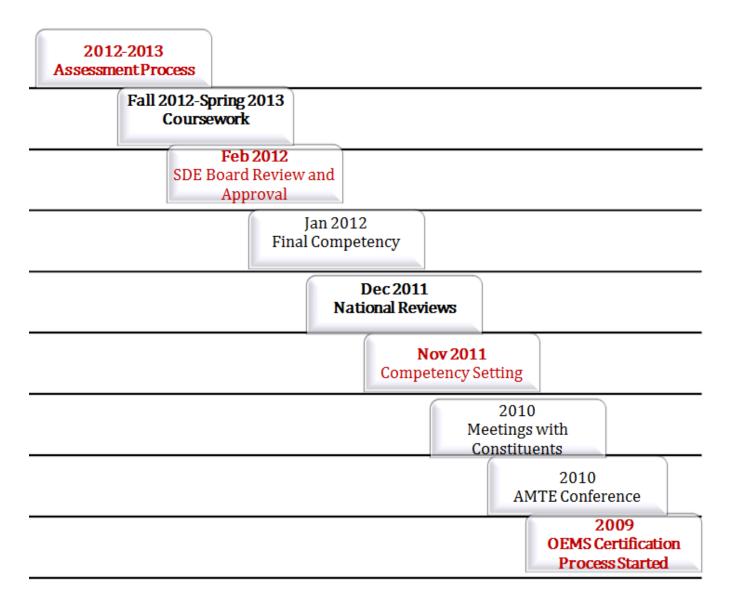
- 1. Valid teacher certification in Elementary Education and/or Early Childhood Education. The OEMS certification cannot be substituted for required certifications in either area. OEMS certification will correspond to the grade level of certification/s already held by the teacher in Early Childhood Education, Elementary Education, or both. For example, if a teacher only holds an Early Childhood certificate and completes the OEMS certification, the OEMS certification will only be issued at the Early Childhood Education grade level.
- 2. Two years of full-time teaching experience in grades **Pre-Kindergarten through Grade 5** at an accredited school under a valid state issued teacher credential;
- 3. Eighteen hours of coursework as outlined below:
  - > 60-70% Pedagogical Mathematics Content Knowledge: Coursework will focus on development of mathematical proficiency as characterized by conceptual

#### **A Certification Process**

understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Research Council, 2001) in the following four domains. Pedagogical Mathematics content shall be consistent with the Common Core State Standards (CCSS).

- · Number Concepts and Operations,
- Algebra and Functions,
- · Geometry and Measurement,
- · Data Analysis and Probability,
- → 30-40% Mathematics Instructional Leadership: Coursework will focus on building effective mathematics leaders who are skilled in planning, developing, and implementing, high quality instruction in mathematics.
- 4. Passing score on the Oklahoma Elementary Mathematics Specialist certification examination.
- 5. Application with the university's recommendation for OEMS certification submitted to the Oklahoma State Department of Education.
- 6. \$25 application fee to accompany the application to the Oklahoma State Department of Education.

#### **OEMS Development Process**



#### \*National Reviewers:

- Dr. Francis (Skip) Fennell, L. Stanley Bowlsbey Professor of Education and Graduate and Professional Studies
- Dr. Maggie McGatha, AMTE, Associate Professor, Mathematics Education, Department of Middle & Secondary Education, College of Education & Human Development, University of Louisville
- Dr. Nicole Miller Rigelman, AMTE, Associate Professor of Mathematics Education Curriculum and Instruction, Portland State University

• Dr. Suzanne Mitchell, Executive Director, Arkansas Science, Technology, Engineering and Mathematics (STEM) Coalition, Arkansas Department of Higher Education

#### **OEMS Certification Examination Structure**

The OEMS certification examination will evaluate candidates on five domains as outlined below:

Domain No.	Domain Title	Credit Hours	
I	Number Concepts and Operations		
II	Algebra and Functions	60-70%	
III	Geometry and Measurement		
IV	Data Analysis and Probability		
V	Mathematics Instructional Leadership	30-40%	
	Total (%)	100%	
	Total (Credit Hours)	18	

#### <u>Pedagogical Content Knowledge for Teaching Mathematics</u>

OEMS professionals must know and understand deeply the mathematics of elementary school as well as how mathematics concepts and skills develop through middle school. This knowledge includes specialized knowledge that teachers need in order to understand and support student learning of elementary mathematics. EMS professionals are expected to have a foundation in pedagogical content knowledge (Ball, Thames, & Phelps, 2008). OEMS are expected to acquire the habits of mind of a mathematical thinker and use mathematical practices such as precision in language, construction and comparison of mathematical representations, conjecturing, problem solving, reasoning, and proving. In Section 1, the domains and corresponding standards are outlined. In Section 2, specific competencies further delineate what the OEMS must know and be able to do.

#### **Section 1: Domains and Standards**

#### **Domain I. Number and Operations: Content, Instruction, and Assessment**

- Pre-number concepts: Non-quantified comparisons (less than, more than, the same), containment (e.g., 5 contains 3), 1-to-1 correspondence, cardinality, ordinality.
- Place value: The structure of place-value notation in general and base-10 notation in particular; how place-value notations efficiently represent even very large numbers, as well as decimals; use of these notations to order numbers, estimate, and represent order of magnitude (e.g., using scientific notation).
- Basic number systems: Whole numbers (non-negative integers), integers, non-negative rational numbers, rational numbers, and real numbers. Relationships among them, and locations of numbers in each system on the number line. What is involved in extending operations from each system (e.g., whole numbers) to larger systems (e.g., rational numbers),
- Multiplicative arithmetic: Factors, multiples, primes, least common multiple, greatest common factor.
- Proportional reasoning and rescaling.
- A comprehensive repertoire of interpretations of the four operations of arithmetic and of the common ways they can be applied.
- Multi-digit calculations, including standard algorithms, mental math, and nonstandard ways commonly created by students; informal reasoning used in calculations.

#### **Domain II. Algebra and Functions Content: Instruction, and Assessment**

- Axioms: Recognize commutativity, associativity, and distributivity, and 0 and 1 as identity elements in the basic number systems; understand how these may be used in computations and to deduce the correctness of algorithms. The need for order-ofoperations conventions.
- Algebraic notation and equations: Literal symbols, as shorthand names for mathematical objects or, in the case of numerical variables, as indicating an unspecified member of some class of numbers (the "range of variation"). The process of substitution of particular numbers into variable expressions. The solution set of an algebraic equation or relation. Transformations of equations (or relations) that do not change the solution set.
- Modeling of problems, both mathematical and "real world," using algebraic equations and relations.
- The concept of a function as defining one variable uniquely in terms of another. Familiarity with basic types of functions, including constant, linear, exponential, and quadratic. Representations and partial representations of functions: Formula, graph, table; or, when the variable is discrete, by recursion.
- Finding functions to model various kinds of growth, both numerical and geometric.
- The concept of a function as defining one variable uniquely in terms of another

#### **Domain III. Geometry and Measurement**

- Visualization: Geometric objects are pictured on a 2-dimensional page; for 3-dimensional objects this requires perspective or projection renderings. Producing and reading such representations calls for special skills, both mathematical and drawing.
- Composing and decomposing: A geometric figure can be assembled by joining together various component figures. Conversely, a geometric figure may be decomposed into pieces, for example decomposing a polygon into an assemblage of triangles.
- Congruence and similarity: *Congruence* is the basic concept of geometric "sameness." *Similarity* has to do with rescaling: Two figures are similar if one of them is congruent to a rescaling of the other. For example, all circles are similar, as are all squares and all isosceles right triangles.
- Geometric measurement: A way of attaching a numerical quantity to a geometric figure. Doing this involves a choice of some standard figure (the "unit") and then the measurement is a kind of ratio of the given figure to the unit, or, put differently, how many copies of the unit does it take to compose the given figure? It follows that if a

#### **A Certification Process**

geometric figure is decomposed, then its measure is the sum of the measures of its components. Changing the unit has the effect of multiplying all measurements by a constant (relating the two units). For example, relating feet to inches, or to meters.

- Common units of geometric measurement:
  - Linear: The unit may be the interval [0, 1] on the number line.
  - > Area: The unit is a unit square.
  - > Volume: The unit is the unit cube.
  - Angle: Draw a unit circle centered at the vertex of the angle, and consider the arc of the circle cut out by the angle. The radian measure of the angle is the length α of that arc. The degree measure of the angle is 360°/2π, i.e. 360 times the fraction of the circumference of the circle formed by the arc.
- Basic geometric figures in each dimension:
  - > Dimension 1: Line segments, arcs of circles;
  - Dimension 2: Polygons, circles;
  - Dimension 3: Polyhedral solids, cylinders, cones, spheres.
- Plane coordinates: How they are introduced, and how they support algebraic expression of geometric objects and relationships. Reciprocally, how they afford geometric interpretation of algebraic relations.
- Transformations: Reflections, rotations, translations, dilations; symmetry and its expression in terms of transformation (e.g., reflection through a line of symmetry).
- Proof: Making and proving conjectures about geometric shapes or relations.
- Elements of these figures, e.g., vertex, edge, face. Properties of regularity and symmetry. Definitions, names, and classification. Various kinds of measurement, and some basic formulas; invariance under congruence, and behavior under rescaling.

#### **Domain IV. Data Analysis and Probability**

- The nature and uses of data: What kinds of questions require data for their answers, and what kinds of data are required? How are relevant data sets created and organized? Designing an investigation, including specification of how the data collected support analysis responsive to the question(s) under investigation.
- Drawing conclusions: Understand which representations best support communication of inferences from data, use probability models when appropriate, and account for variability. Understand the limits of generalizability due to non-randomness of a sample population.
- Distinguish categorical (discrete) data (e.g., gender, favorite ice cream flavor) from measurement (continuous) data.

### **A Certification Process**

- Appropriate types of representation of data, and what they afford: For categorical data, relative frequencies. For measurement data, displays of shape, center, and spread.
- Basic concepts of probability and ways to represent them; making judgments under conditions of uncertainty; measuring likelihood; becoming familiar with the concept of randomness.

#### **Domain V. Mathematics Instructional Leadership**

OEMS professionals need to be prepared to take on collegial non-evaluative leadership roles within their schools and districts. They must have a broad view of many aspects and resources needed to support and facilitate effective instruction and professional growth. They must be able to:

- Use professional resources such as professional organization networks, journals, and discussion groups to be informed about critical issues related to mathematics teaching and learning, e.g., policy initiatives and curriculum trends.
- Select from a repertoire of methods to communicate professionally about students, curriculum, instruction, and assessment to educational constituents—parents and other caregivers, school administrators, and school boards.
- Plan, develop, implement, and evaluate professional development programs at the school and district level and support teachers in systematically reflecting and learning from practice. Evaluate educational structures and policies that affect students' equitable access to high quality mathematics instruction, and act professionally to assure that all students have appropriate opportunities to learn important mathematics.

#### **A Certification Process**

#### **Section 2: Competencies**

# **DOMAIN I: Number Concepts and Operations: Content, Instruction, and Assessment**

<u>Competency 001:</u> The OEMS understands the structure of number systems, student's development of a sense of quantity, and the relationship between quantity and symbolic representations.

- A. Analyzes the structure of numeration systems and the roles of place value and zero in multiple base systems.
- B. Analyzes and differentiates a variety of models that student will use to represent numbers (e.g., patterns, sets, number lines, base ten blocks, diagrams, shaded regions, fraction strips).
- C. Compares and contrasts different representations of equivalent rational numbers.
- D. Explains and applies models to show how some situations have no solution and/or multiple solutions in the real number system.
- E. Explains and applies students' sense of quantity and estimation for the real numbers and the relationship between quantity and estimation.
- F. Analyzes pre-number concepts and connects those concepts to number systems including non-quantified comparisons (less than, greater than, equal), containment (e.g., 5 contains 3), 1-to-1 correspondence, cardinality, ordinality.
- G. Compares and contrasts numeration systems (additive, multiplicative, ciphered, positional-place value).
- H. Compares and contrasts multiple representations of numbers in real-world situations.
- I. Compares and contrasts the characteristics of the sets of whole numbers, integers, rational numbers, real numbers and complex numbers (e.g., properties of operations, inverse elements, density) so that student's instruction is both developmentally appropriate and vertically aligned.

<u>Competency</u> <u>002:</u> The OEMS understands number operations and computational algorithms, and students' development of these concepts with a goal of computational fluency.

A. Analyzes, justifies, and connects relationships among number properties and a variety of algorithms involving the basic operations with real numbers and complex numbers.

#### **A Certification Process**

- B. Justifies procedures used in algorithms for the basic operations with integers, rational numbers and real numbers, and analyzes error patterns that may occur in their application.
- C. Recognizes and analyzes a variety of algorithms for the basic operations with whole numbers, fractions, and decimals and helps students understand innovative self-created methods (e.g., rectangular arrays, partitioning, composing, decomposing).
- D. Analyzes the possibilities and limitations of different algorithms, including why they may work in some situations and not others.
- E. Understands the progression of student development from concrete to pictorial to symbolic representations of the basic operations with real numbers, and recognizes the importance of teaching these concepts and relationships in real-world situations, using technology where appropriate.
- F. Analyzes error patterns that often occur when students use algorithms to perform operations.

<u>Competency 003:</u> The OEMS understands elementary number theory and ways in which students use numbers to model and solve problems.

- A. Analyzes the relationships between different forms of multiplicative arithmetic (e.g., factors, multiples, primes, composites, least common multiple, greatest common factor) and their connections to other domains of mathematics (e.g., fractions, exponents, algebra).
- B. Applies properties of the real numbers to solve a variety of theoretical and applied problems.
- C. Applies knowledge of place value and other number properties to develop students' strategies for mental mathematics, computational estimation, and rounding.
- D. Creates innovative problems in a variety of real-world situations that involve integers, fractions, decimals and percents.
- E. Analyzes and evaluates informal reasoning and non-standard, inventive strategies created by students.
- F. Evaluates the impact of the characteristics of the sets of whole numbers, integers, rational numbers and real numbers (e.g., property of operation, inverse elements, fraction density) on students' understanding of operations.

<u>Competency 004:</u> The OEMS implements effective instructional design strategies to guide, evaluate, and improve all students' learning of number concepts and operations.

### **A Certification Process**

- A. Use questions to effectively probe students' mathematical understanding and make productive use of their responses with respect to number concepts and operations.
- B. Supports students' learning of appropriate technical language associated with mathematics and their ability to communicate effectively.
- C. Designs challenging and engaging problem-solving tasks that develop numberconcept content knowledge as well as students' critical and analytical reasoning capacities.
- D. Designs, analyzes, and uses assessment results from various instruments (e.g. diagnostic, formative, summative) to plan, inform, and adjust instruction.
- E. Recognizes and uses the vertical alignment of number concepts across grade levels to plan instruction based on state standards.
- F. Recognizes how to improve equity for all students in mathematics instruction through reflection on one's own attitudes, expectations and teaching practices with respect to number operations and concepts.
- G. Designs, selects and adapts worthwhile mathematical tasks and sequences of examples that support developing conceptual understanding, computational fluency, and precision.
- H. Analyzes evidence of students' current understanding of number concepts to select strategies to help students move from informal to formal knowledge.
- I. Diagnoses mathematical misconceptions and errors and designs appropriate interventions.
- J. Evaluates and applies established research evidence on how all students learn and use number concepts.
- K. Evaluates and applies established research evidence on using technology to effectively support student learning of number concepts.

#### **DOMAIN II: Algebra and Functions: Content, Instruction, and Assessment**

<u>Competency 005:</u> The OEMS understands and uses mathematical reasoning to identify, extend, and analyze patterns, as well as the relationships among variables, expressions, equations, relations, and functions to effectively support student development and learning of these concepts.

The OEMS helps students' and teachers to:

- A. Use inductive reasoning to identify, extend, generalize, and create patterns with numbers, concrete models, geometric figures, tables, graphs and algebraic expressions.
- B. Use a variety of number patterns to explore number properties.
- C. Formulate implicit and explicit rules to describe and construct sequences verbally, numerically, graphically, and symbolically.
- D. Give appropriate justification of the manipulation of algebraic expressions, equations and inequalities in mathematical and real-world problem-solving situations.
- E. Understand the concept and analyze the attributes of functions and relations (e.g. domain, range, one-to-one functions, inverse functions) and their multiple representations.
- F. Incorporate a variety of strategies, including tools and appropriate technology, to explore number patterns and properties (e.g., fact families, number charts, multiplication by powers of 10).
- G. Explore how problem-solving situations translate into expressions and equations.

<u>Competency 006:</u> The OEMS understands and uses linear functions to model and solve problems, using a variety of methods to effectively support student development and learning of algebraic concepts.

- A. Demonstrates an understanding of the concept of linear functions and systems of linear equations and inequalities by using concrete models, tables, graphs, or other verbal and symbolic representations.
- B. Demonstrates an understanding of the connections among linear functions, proportions, and direct variation.
- C. Analyzes the relationships among a linear function, its average rate of change, and its graph.
- D. Demonstrates an understanding of the characteristics of linear models and the advantages/disadvantages of using a linear model in a given situation.

### **A Certification Process**

- E. Uses transformations to illustrate properties of linear functions to solve problems and to predict the effects of transformations on the graphs of linear functions.
- F. Justifies the manipulation of algebraic expressions and equations.
- G. Uses graphic calculators or similar technologies to investigate and apply the connections between linear functions and their concrete models, tables, graphs and symbolic and verbal representations.
- H. Compare and contrast the characteristics of linear and nonlinear functions and their corresponding models, tables and graphs.
- I. Explores problems using concrete, geometric, tabular, graphic and algebraic methods to develop students' understanding of linear functions.

<u>Competency 007:</u> The OEMS implements effective instructional design strategies to guide, evaluate, and improve all students' learning of patterns, algebra, and functions.

- A. Use questions to effectively probe students' mathematical understanding and make productive use of their responses with respect to patterns, algebra and functions.
- B. Supports students' learning of appropriate technical language associated with patterns, algebra and function and their ability to use that language to communicate effectively.
- C. Designs challenging and engaging problem-solving tasks that develop algebraic content knowledge as well as students' critical and analytical reasoning capacities.
- D. Designs, analyzes, and uses assessment results from various instruments (e.g. diagnostic, formative, summative) to plan, inform, and adjust instruction.
- E. Recognizes and uses the vertical alignment of patterns, algebra, and function concepts across grade levels to plan instruction based on state standards.
- F. Recognizes how to improve equity for all students in mathematics instruction through reflection on one's own attitudes, expectations and teaching practices with respect to algebraic concepts.
- G. Designs, selects and adapts worthwhile mathematical tasks and sequences of examples that support developing conceptual understanding, computational fluency and precision.
- H. Analyzes evidence of students' current understanding of algebraic concepts to select strategies to help students move from informal to formal knowledge.
- I. Diagnoses mathematical misconceptions and errors and designs appropriate interventions.
- J. Evaluates and applies established research evidence on how all students learn and use patterns, algebra, and functions.

**K.** Evaluates and applies established research evidence on using technology to effectively support student learning of patterns, algebra, and functions.

#### **DOMAIN III: Geometry and Measurement: Content, Instruction, and Assessment**

<u>Competency 008:</u> The OEMS understands the development of measurement as a process and ways in which students use measurement to model and solve problems.

- A. Understands the selection, quantifications, and comparison of the attributes to be measured by using and communicating appropriate units (standard and nonstandard).
- B. Uses and justifies conversions within and between measurement systems.
- C. Illustrates and explains the use of numbers and units of measurement for quantities such as length, perimeter, circumference, angles, area, volume, temperature, time, percent, speed, and acceleration.
- D. Illustrates the composition and decomposition of geometric figures in order for student to measure and understand the attributes of the figures.
- E. Describes the precision of measurement using methods of approximation and estimating the effects of error on measurement.
- F. Applies measurement, including proportional reasoning, to solve real-world problems.
- G. Support students' learning of dimensional analysis to derive units and formulas in a variety of situations and to find and evaluate solutions to problems.

<u>Competency</u> **009:** The OEMS understands the basic concepts and applications of Euclidean geometry, and effectively supports students' development of these concepts.

- A. Understands and demonstrates concepts and properties of and relationships among one-, two-, and three-dimensional objects.
- B. Describes and justifies geometric constructions made using a compass and straight edge, other manipulative materials, and appropriate technologies to guide students in their appropriate usage.
- C. Uses logical reasoning to analyze and prove geometric relationships.
- D. Analyzes concepts of Euclidean geometry and helps students solve problems involving one-, two-, and three-dimensional objects.

#### **A Certification Process**

**Competency 010:** The OEMS understands transformational and coordinate geometry and connects geometry with other topics in the mathematics curriculum to effectively support students' development of these concepts.

- A. Describes and justifies, using visualization, geometric modeling and construction, and spatial reasoning to investigate and describe shape in terms of dimension, direction, orientation, and perspective.
- B. Uses translations, reflections, glide-reflections and rotations to demonstrate congruence and to explore the symmetries of figures.
- C. Uses symmetry to describe, show, and analyze tessellations and how they can be used to illustrate geometric concepts, properties, and relationships.
- D. Analyzes the relationship among geometrics, fractional, and algebraic representations and uses this information to solve problems.
- E. Applies concepts and properties of lines in the coordinate plane to explore the properties of geometric figures and solve problems.
- F. Uses dilations (expansions and contractions) to illustrate, and solve problems involving similar figures and proportionality.

<u>Competency 011:</u> The OEMS implements effective instructional design strategies to guide, evaluate, and improve all students' learning of geometry and measurement.

- A. Use questions to effectively probe students' mathematical understanding and make productive use of their responses with respect to geometry and measurement.
- B. Supports students' learning of appropriate technical language associated with geometry and measurement and their ability to use that language to communicate effectively.
- C. Designs challenging and engaging problem-solving tasks that develop geometry and measurement content knowledge as well as students' critical and analytical reasoning capacities.
- D. Designs, analyzes, and uses assessment results from various instruments (e.g. diagnostic, formative, summative) to plan, inform, and adjust instruction.
- E. Recognizes and uses the vertical alignment of geometry and measurement concepts across grade levels to plan instruction based on state standards.
- F. Recognizes how to improve equity for all students in mathematics instruction through reflection on one's own attitudes, expectations and teaching practices with respect to geometry and measurement concepts.

#### **A Certification Process**

- G. Designs, selects and adapts worthwhile mathematics tasks and sequences of examples that support developing conceptual understanding, computational fluency and precision.
- H. Analyzes evidence of students' current understanding of geometry and measurement concepts to select strategies to help students move from informal to formal knowledge.
- I. Diagnoses mathematical misconceptions and errors and designs appropriate interventions.
- J. Evaluates and applies established research evidence on how all students learn and geometry and measurement.
- K. Evaluates and applies established research evidence on using technology to effectively support student learning of geometry and measurement.

#### DOMAIN IV: Data Analysis and Probability: Content, Instruction, and Assessment

<u>Competency 012:</u> The OEMS understands the theory of probability and the ways in which students apply it to real-world.

- A. Explains the connections between data collection, experiments and simulations to concepts of probability.
- B. Justifies an appropriate sample space that models a situation to draw inferences about a population.
- C. Analyzes models and applies concepts of probability.
- D. Applies knowledge of the use of probability to make observations about and draw conclusions from data, and to describe the level of confidence in the conclusion.

<u>Competency 013:</u> The OEMS understands how to collect, describe, display, and draw appropriate inferences from data to effectively support students; development of these skills.

- A. Compares and translates data in a variety of formats (e.g., tables, frequency distributions, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts, line Plot).
- B. Applies knowledge of designing, conducting, analyzing and interpreting experiments to investigate real-world problems.
- C. Differentiates between types of data and the appropriate representation (e.g., relative frequencies for categorical data and displays of shape, center, and spread for measurement data).
- D. Draws conclusions about which representations best support communication of inferences from data.
- E. Uses probability models when appropriate, accounts for variability and understands the limits of generalization due to non-randomness of a sample population.
- F. Integrates and interprets probability and statistics within and outside of mathematics, including misleading or misinterpreted data.

<u>Competency 014:</u> The OEMS implements effective instructional design strategies to guide, evaluate, and improve all students' learning of data analysis and probability.

A. Use questions to effectively probe students' mathematical understanding and make productive use of their responses with respect to data analysis and probability.

#### **A Certification Process**

- B. Supports students' learning of appropriate technical language associated with data analysis and probability and their ability to use that language to communicate effectively.
- C. Designs challenging and engaging problem-solving tasks that develop data analysis and probability content knowledge as well as students' critical and analytical reasoning capacities.
- D. Designs, analyzes, and uses assessment results from various instruments (e.g. diagnostic, formative, summative) to plan, inform, and adjust instruction.
- E. Recognizes and uses the vertical alignment of data analysis and probability concepts across grade levels to plan instruction based on state standards.
- F. Recognizes how to improve equity for all students in mathematics instruction through reflection on one's own attitudes, expectations and teaching practices with respect to data analysis and probability.
- G. Designs, selects and adapts worthwhile mathematics tasks and sequences of examples that support developing conceptual understanding, computational fluency, and precision.
- H. Analyzes evidence of students' current understanding of data analysis and probability concepts to select strategies to help students move from informal to formal knowledge.
- I. Diagnoses mathematical misconceptions and errors and designs appropriate interventions.
- J. Evaluates and applies established research evidence on how all students learn and data analysis and probability.
- K. Evaluates and applies established research evidence on using technology to effectively support student learning of data analysis and probability.

#### **DOMAIN V: Mathematics Instructional Leadership**

<u>Competency 015:</u> The OEMS understands how to communicate and collaborate with educational stakeholders to facilitate implementation of appropriate, standards-based mathematics instruction, guide instructional improvement, and foster a school culture rich in mathematics.

- A. Collaborates with others to teach mathematics in the context of other content areas and to effect positive change in the culture of the school mathematics program and mathematics instruction.
- B. Evaluates the results of school, individual student, classroom, district, state and national assessments to monitor mathematics programs, prioritize needs and plan strategies for improvement.
- C. Uses professional resources such as organization networks, journals, and discussion groups to stay current regarding critical issues related to mathematics teaching and learning.
- D. Selects appropriate methods to communicate professionally about students, curriculum, instruction, and assessment to educational stakeholders.
- E. Understands educational structures and policies that affect students' equitable access to high quality mathematics instruction.
- F. Plans, develops, implements, and evaluates professional development through mentoring, coaching, and consultation with colleagues to facilitate appropriate, standards-based mathematical instruction, and makes instructional decisions supported by established research.